

Annex 3: Formulas and Approaches used to calculate APRs

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The purpose of the Annual Percentage Rate (APR)

The true price of a loan includes not only interest but other charges required by the lender as well as other techniques that influence the amount of money the client actually has and the amount of time the client has use of that money. Because of these multiple factors, as well as differences in interest calculation methods, comparing the pricing of different loan products can be very challenging. The Annual Percentage Rate (APR) is used to express the true price as a standard measure that allows for the comparison of credit charges among different loan products.

To understand how to calculate the APR, it is necessary to first understand the time value of money and how to calculate the present value of future monetary amounts.

The Time Value of Money

The principle of the time value of money assumes it is preferable to receive a given amount of money today rather than at some point in the future, everything else being equal. The time value of money reflects the ability to earn a return on investment.

As interest is the charge for the use of money over time, it incorporates the concept of the time value of money. Thus, not only the amount the borrower pays, but also the timing of those payments affects the overall cost to the borrower.

As a simple example, consider the timing of when interest is paid on a loan. The MFI will be interested in receiving the interest payments sooner rather than later. On a simple loan of \$100 for 12 months, with the principal due in a single payment in 12 months, the MFI will prefer that the interest be paid monthly rather than accrued. Once the MFI receives interest payments, it can use the money to make additional loans on which it would earn interest. The client, on the other hand, would prefer to pay all the interest in one single payment at the end of the loan, because then she could then use that money in her business for a longer period of time.

Calculating Future and Present Value of Money

As money is considered to have different values at different points in time, a loan amount can have both a present value and a future value.

Because time comes in different units, in all the formulas used in APR calculations, it is essential to use consistent units. All time periods “*t*” must be of equivalent duration, e.g., a week, a month, a year. Also, the interest rate “*i*” must be valued for the same time duration. APR formulas define the time interval as the shortest period of time where a monetary transaction (generally a repayment) takes place, e.g., a week, even if a transaction does not take place each and every week.

To begin with the simplest formula, the **future value A** of an amount **PV**, when compounded at the interest rate i for t time intervals is:

$$A = PV * (1 + i)^t$$

Again, i is the interest rate for each time interval t .

Example 1: Consider this to be a savings deposit of \$ 100 (PV), earning an interest rate of 1 % per month, for 12 months.

$$A = 100 * (1+0.01)^{12} = 112.68$$

Rather than being worth only \$112 in 12 months, the savings will be worth \$ 112.68 in 12 months due to compounding, i.e., earning interest in future periods upon interest earned in previous periods.

If the product were advertised as paying 12% interest at the end of 12 months, then the APR would simply be 12%. In this example, the time interval t is one year, and the interest rate i is 12.0%

$$A = 100 * (1+0.12)^1 = 112.00$$

Rearranging the formula, we can see that the **present value** is:

$$PV = \frac{A}{(1 + i)^t}$$

The present value of any amount equals the future value discounted by the interest rate and amount of time.

Example 2: In 12 months, the client is to pay \$ 126.82. Given an interest rate of 2 % per month, the present value of that \$ 126.82, is:

$$PV = 126.82 / (1+0.02)^{12} = \$ 100$$

In other words, that payment of \$126.82 in the future is equivalent to the client having paid \$100 today.

The previous formulas calculate future and present value for a single monetary transaction. The following formula adapts the present value formula to give the cumulative present value of a string of future cash flows:

$$PV = \sum_{t=0}^n \frac{A}{(1 + i)^t}$$

For a period of 12 monthly transactions this is the equivalent of:

$$PV = \frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \dots$$

The Discount Rate Method of Calculating APR

The APR is defined as the interest rate that would make the present value of the loan received by the client equal to the present value of the installments paid by the client. In other words, the above Present Value formula is applied to both advances "A" (i.e., the loan disbursements received by the client) and to the installments paid by the client "P".

This is shown in the following formula, called the Discount Rate Method:

$$\sum_{k=1}^m \frac{A_k}{(1+i)^{q_k}} = \sum_{j=1}^n \frac{P_j}{(1+i)^{t_j}}$$

Where

- A_k: amount of the kth advance
- q_k: number of full unit-periods from the beginning of the term of the transaction to the kth advance
- m: number of advances
- P_j: amount of the jth payment
- t_j: number of full unit-periods from the beginning of the term of the transaction to the jth payment
- n: number of payments
- i: percentage rate of finance charge per unit-period, expressed as a decimal equivalent

We can apply this formula to a simple loan example of \$1,000 for 12 months with monthly interest of 1%, and equal monthly installments. The nominal values of the disbursement and repayment schedule can be seen in the two left columns of this table:

Period	Nominal Values				Discounted Values	
	Disburse	Repay	Divisor	%	Disburse	Repay
0	1,000.00	0.00	1.00000		1,000.00	0.00
1	0.00	88.85	1.01000	1.00%	0.00	87.97
2	0.00	88.85	1.02010	1.00%	0.00	87.10
3	0.00	88.85	1.03030	1.00%	0.00	86.24
4	0.00	88.85	1.04060	1.00%	0.00	85.38
5	0.00	88.85	1.05101	1.00%	0.00	84.54
6	0.00	88.85	1.06152	1.00%	0.00	83.70
7	0.00	88.85	1.07214	1.00%	0.00	82.87
8	0.00	88.85	1.08286	1.00%	0.00	82.05
9	0.00	88.85	1.09369	1.00%	0.00	81.24
10	0.00	88.85	1.10462	1.00%	0.00	80.43
11	0.00	88.85	1.11567	1.00%	0.00	79.64
12	0.00	88.85	1.12683	1.00%	0.00	78.85
	1,000.00	1,066.19		12.7%	1,000.00	1,000.00

It can be seen that the client receives \$1,000 and pays back a total \$1,066.19 over 12 months. The center column shows the denominator of the PV formula when the monthly interest rate is 1.0%. The divisor for Period 0 is $(1.01)^0 = 1.000$. The divisor for Period 1 is $(1.01)^1 = 1.01$. The divisor for Period 2 is $(1.01)^2 = 1.0202$. Note that it is not 1.0200 because of the compounding factor. And the process continues.

The right hand columns divide the disbursements and repayments for each period by the period's divisor and result in the discounted present value of those amounts. In summing up the total discounted present values of the disbursements and repayments, one sees that they are both equal to \$1,000, as required by the Discount Rate Method equation.

Thus, in this example "i", the period interest rate is 1.0%. In this case, the cost of the loan was a straightforward 1.0% per month calculated on the declining balance. Thus, the Discount Rate Method also gives a figure of 1.0% per month because this formula also tells you what the declining balance interest rate would be. In more complicated situations, involving fees or different interest rate calculation methods, the Discount Rate Method could be used to determine the equivalent declining balance interest rate that is equivalent to the disbursement and repayment cash flows.

In studying this more carefully, one sees that we know all variables in the Discount Rate formula except for *i*. We are to use an iterative process where different values of "i" are tried until the sums of both disbursements and repayments are equal. For example, trying *i* = .9% gives a discounted present value of \$1,006.35 for the repayment stream. Trying *i* = 1.1% gives \$993.71. With *i* = 1.0%, the value is exactly \$1,000 and equal to the discounted present value of the disbursement stream.

There are two common approaches that have been legislated using the Discount Rate Method: the US method and the EU method. The difference between these two methods is how to now convert the *per unit period* interest rate figure into an *annual* figure. In other words, the interest rate in the above example has been determined to be 1% *per month*. How should that value be converted to an *Annual Percentage Rate*? The US method calculates what would be called a *nominal* annual rate, while the EU method determines an *effective* annual rate.

The nominal annual rate is the annual interest rate without taking into account the effects of compounding. It is simply the period interest rate times the number of periods in a year:

Example: 1.0% per month
Nominal rate: $1.0\% * 12 = 12.0\%$

In other words, if the lender tells you the interest rate is 1%, calculated monthly on a declining balance, then the US-APR of that loan is 12.0%.

However, the effective annual rate considers compounding factors. It is therefore calculated as:

Effective rate: $(1.01)^{12} - 1 = 12.68\%$

The EU-APR for this loan would legally be stated as 12.68%. By experimentation, one will find that if a loan is quoted with an EU-APR of 12.0%, with interest calculated monthly, the monthly interest rate charged must be not 1.0%, but 0.95%. The effective rate formula shows that:

Effective rate: $(1.0095)^{12} - 1 = 12.00\%$

The difference between APRs quoted with the US formula and the EU formula diverge more as the number of compounding periods in the year increase, i.e., loans with interest calculated quarterly will have APRs that are close in both formulas, but when interest is calculated weekly, or even daily, the two APRs can give quite different figures.

